Transactive Control of Electric Railways Using Dynamic Market Mechanisms

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Abstract—Electricity demand of electric railways is a relatively unexplored source of flexibility in demand response applications in power systems. In this paper, we propose a transactive control based optimization framework for coordinating the power grid network and the train network. This is accomplished by coordinating dispatchable distributed energy resources and demand profiles of trains using a two-step optimization. A railway based dynamic market mechanism (rDMM) is proposed for the dispatch of distributed energy resources (DER) in the power network along the electric railway using an iterative negotiation process, and generates profiles of electricity prices, and constitutes the first step. The train dispatch attempts minimize the operational costs of trains that ply along the railway, while subject to constraints on their acceleration profiles, route schedules, and the train dynamics, and generates demand profiles of trains and constitutes the second step. The rDMM seeks to optimize the operational costs of the underlying DERs while ensuring power balance. Together, they form an overall framework that yields the desired transactions between the railway and power grid infrastructures. This overall optimization approach is validated using simulation studies of the Southbound Amtrak service along the Northeast Corridor (NEC) in the United States, which shows a 25% reduction in energy costs when compared to standard trip optimization based on minimum work, and a 75% reduction in energy costs when compared to the train cost calculated using a field dataset.

Index Terms—Power Grid, Train Network, Renewable Integration, Railway Dispatch, Train Dispatch, Social Welfare, Trajectory Optimization.

I. INTRODUCTION

A. Motivation

Modern electric trains can both demand power from their traction system for locomotion and inject power back into the electricity network through regenerative braking, virtually enabling them to store electricity in the form of kinetic energy [1]. The power profile of a train along a route is in many cases determined by the conductor based on training and experience, attempting to meet a given schedule with little regard to the varying electricity price along the route. In this paper, we propose an alternate operation methodology that consists of coordination of train schedules and the dispatch of rail-side distributed energy resources (DERs) and leads to a determination of prices and schedules of power consumption for the trains and power generation for the DERs.

Together, this coordinated operation is shown to minimize the overall electricity cost incurred by the trains. As this coordination occurs through a transactive framework between the train dispatch and the dispatch of railway agents such as DERs, this leads to a transactive control of the two interconnected systems of train network and the power grid network, similar to the transactive controller in [2], [3]. Additionally, this operation methodology accommodates complex traction and train infrastructure ownership structures. This can include systems where multiple train operators traverse tracks owned by various railway operators that can in turn be customers of numerous electric utilities and rail-side DER operators.

B. Literature Review

Related work that has addressed trip optimization in rail networks can be found in [4], [5], [6], [7], and [8]. Reference [4] provides a summary of the trajectory planning problem in railway systems. Optimal trajectory planning for electric railways can be found in chapters 3 and 4 of [5], where pseudospectral methods are used to determine optimal railway operation based on models of train dynamics. This work builds upon the work minimization literature developed by [6] and [7]. Finally, [8] develops a control system to reduce fuel use in freight locomotives. In all of these lines of research, the overall objective is to minimize energy use, or work done by the train, rather than the cost of the electricity to the infrastructure manager, an important component of our proposed scheme.

A major driver that allows the proposed transactive coordination framework between the train-network and the power-network is the transformation of the latter in recent years. This has been due to the explosive growth of renewable energy sources [9] and demand response [10], collectively denoted as DERs. The increasing footprint of these resources enables them to be dispatchable, and to enter into a transacational framework where incentive information (pricing) and commitment decisions (quantity) can be iteratively exchanged and arrived at an optimal solution. This iterative transacational framework is denoted as Dynamic Market Mechanisms (DMM), and has been explored in [11]–[14]. The origins of
transactive control are very much rooted within the energy application realm, as the ideas of using an incentive signal to alter the behavior of demand-side customers in power systems can be traced back to Schewpe’s 1980 paper on homeostatic utility control [3]. A large-scale demonstration of this concept can be found in [2] where electric systems with high renewable adoption were considered and shown to meet demand reduction objectives. The DMM-related results in [11] show that DERs can engage in market-transactions at the tertiary control level and ensure grid objectives through a hierarchical framework, with asymptotic stability, which results in convergence within a region of attraction. Exactly how these market mechanisms can be integrated into a real-time market and a regulation market together with secondary control based AGC was explored in [14]. Focus was placed on demand response compatible assets in [12] which provides sufficient conditions for convergence. The implications of DMM in the context of a combined heat and power microgrid was explored in [15], and [16].

Key differences with other demand response applications are tied to the spatio-temporal constraints of the overall grid-railway optimization problem. Other DR applications are posed with assets that are spatially constant, whereas trains enter and depart area control centers tied to the electric infrastructure that powers them. From the optimization lens, the force balance constraint of train dispatch introduces a conservation equation that is not common in DR.

C. Contributions and Paper Organization

In this paper, we will enable the coordination of various DERs in a power network that are located along the railway network through the use of a DMM, and denote it as rDMM. The main challenge in the design of the proposed transactive framework is to coordinate the objectives of the two networks. The points of intersection between these two networks is the need to optimize operational costs and the need to ensure physics-based constraints such as power balance, capacity and operational limits, and train kinematics. A combined optimization problem subject to all underlying constraints can be posed and used to determine the train schedules and prices, but can prove to be quite intractable due to the complex nature of the constraints, space-dependent and time-dependent constraints with various intractable coupling mechanisms. We therefore adopt a two-step approach where the first consists of railway dispatch of schedules and electricity prices for a given train-demand profile, and the second consists of train dispatch which determines the train schedules for a given electricity-price profile.

The railway dispatch determines, along each section of the track, the electrical output of each generator, the output of all storage assets, and the output of all cogeneration assets for a given set of profiles of power demand from trains and renewable generation. The train dispatch solves the trajectory optimization problem, i.e. the velocity profiles of the trains, through an energy cost minimization subject to acceleration limits and kinematic constraints. Our thesis in this paper is that such a two-step optimization can enable an effective coordination between the power grid network and railway network. In particular, we will show that the two-step optimization will lead to significant reduction in energy costs through simulation studies. While the methods utilized for solving this two-step optimization are fairly straightforward, the main contribution of the paper lies in the novelty of the proposed approach for trip optimization in electric railway networks. The second contribution of this paper is a demonstration through a case study of the Amtrak service along the Northeast Corridor in the United States and composed of multiple sections and DER topologies. To our knowledge, such a transactive approach, which can be viewed as Demand Response using the flexibility in power consumption of trains, has not been suggested thus far in the literature except for [17] and [18], where we presented preliminary results using this approach.

The remainder of this paper is organized as follows. Section II describes the problem faced by the railway and train operators in scheduling DERs and trains along the railway system. In Section III we break out the railway dispatch, i.e. dispatch of the generators and other assets along the railway track based on the estimated renewable generation, traction electric demand and electric and thermal loads for each section of the electric railway. Section IV establishes the dynamic model of an electric train and formulates the energy cost minimization problem that needs to be solved by each train traveling along the electric railway. Section V describes the integrated transactive control methodology that iterates between the Railway Dispatch and the Train Dispatch solutions to determine the price signals and corresponding dispatch profiles of the agents and trains that minimize the operational costs of the entire system. Section VI presents a case study of the Amtrak service and validates the proposed transactive controller. Realistic accommodation of data has been carried out in this case study including incorporation of the actual electricity prices from the wholesale market, actual load profiles, realistic train data, and renewable energy profiles available in public database. Using this case study, we compare our approach with both the current train profile using field data and an optimization framework based on minimization of work. Concluding remarks and future research extensions are discussed in Section VII.

II. PROBLEM FORMULATION

Electric railway systems can be owned and operated by multiple parties. In some systems, the track and electric system that powers the trains are owned by an entity that charges a fee for the utilization of their facilities. This entity is typically responsible for the maintenance of the track, procuring the electric power to feed the trains, dispatching rail-side DERs and controlling traffic along the system. These entities are commonly known as Railway Operators. We will denote Train Operators as those who are in charge of the use of the track based on their projected train schedules, maintain and dispatch the trains and pay usage fees to the various Railway Operators along the tracks used by their trains. The problem that we consider in this paper is a combined optimization of both the railway dispatch and the train dispatch, with Railway Operator and Train Operator acting as the interface between the grid
A problem formulation for energy procurement and dispatch

B. Overall Constrained Optimization Problem

Commercial companies including electric railways. Established and growing energy procurement mechanisms for DERs, seeking to provide energy services to the Railway Operator. Competitive retail sales of electricity from power generators could access wholesale markets for electricity supply. Additionally, we assume that third-parties could invest in rail-side generators as well as grid-tied systems. In some scenarios, electrical connections to a large-scale electricity distribution or transmission system are complemented with distribution lines that travel along the rail, and can be designed such that they improve the reliability of the traction system by providing redundant power supplies to the traction substations feeding the traction system. These various architectures have a direct impact on the flexibility of the demand of the train and therefore a possible energy cost reduction.

A. Traction System Preliminaries

A wide range of traction system architectures and technologies have been developed for electrified railway systems, across low frequency (DC, 16.6Hz, 20 Hz and 25Hz) as well as industrial frequency (50Hz and 60Hz) networks which are powered by overhead and third rail traction distribution systems. Additionally, electric trains have been both powered by rail-side generators as well as grid-tied systems. In some scenarios, electrical connections to a large-scale electricity distribution or transmission system are complemented with distribution lines that travel along the rail, and can be designed such that they improve the reliability of the traction system by providing redundant power supplies to the traction substations feeding the traction system. These various architectures have a direct impact on the flexibility of the demand of the train and therefore a possible energy cost reduction.

A typical architecture of energy procurement and dispatch that occurs along the electric railway is provided in Fig. 1 and will be adopted for the discussions in this paper. This architecture is typically adopted in deregulated electricity markets such as Amtrak in the Northeastern United States, and will be used in the case study presented in Section VI. That is, we assume that Railway Operators face delivery charges as distribution-level customers of the various electric utilities that own and operate the distribution systems that feed railway facilities. However, due to the energy requirements and the regulatory setting in which electric railways procure energy, Railway Operators could access wholesale markets for electricity supply. Additionally, we assume that third-parties could invest in rail-side DERs, seeking to provide energy services to the Railway Operator. Competitive retail sales of electricity from power marketers [19] and on-site Power Purchase Agreements (PPAs) offered by Energy Services COmpanies (ESCOs) [20] are both established and growing energy procurement mechanisms for commercial companies including electric railways.

B. Overall Constrained Optimization Problem

With the architecture chosen as in Fig. 1, we now introduce a problem formulation for energy procurement and dispatch for optimal grid-railway interaction. A first component of this problem formulation is the Area Control Centers (ACCs) (see Fig. 1) managed by Railway Operators. An ACC is charged with serving the electric railway traction system, limited to a portion of contiguous electric railway with the property that marginal injections or demands for power have the same cost to the operator for all time. Along this portion of the track, all rail-side DERs and trains that interface with the traction system are enabled to provide price and quantity information regarding their dispatch, and are compensated and charged based on their actions by the ACC. A Railway Operator manages each ACC with the objective of reducing overall operational costs while maximizing the value of the DERs along the track and meeting all thermal and electric loads in the system. That is, the overall objective of the railway dispatch is to schedule energy resources, which includes trains, so as to minimize the cost of energy resources and trains along the railway system. Such an optimization has to be carried out subject to the various constraints of the grid and railway networks.

More formally, the underlying problem is the dispatch of electric railway power systems across N ACCs in an optimal manner. This includes the electric traction loads of L trains, with power profiles denoted as $P_{t_n}^d(t_n)$ for the electric profile of train l in ACC over the time interval $t_n \in [t_{n,0}, t_{n,1}]$. The Railway Operator is charged for its electric loads at a rate $\lambda_n(t)$. On the generation side, we consider $D_n$ dispatchable generators (including DERs and electric utility imports) at each ACC with electric and thermal power profiles $P_{th,n}^d$ and $P_{th,n}^t$. For compactness, we will use $P_{th,n}^d$, a tuple of the electric and thermal generation for each generator d. Dispatchable agents incur a cost $C_{th,n}^d (P_{th,n}^d)$, which is private to their owner and operator.

All other electric and thermal power demand in the electric railway system, such as the thermal conditioning and lighting loads at passenger stations, are considered price-inelastic and are included within the power balance constraints of the problem for each ACC as $P_{n}^e$ and $P_{n}^th$ but are not included...
in the objective function due to their fixed nature.

Given that the utility of the trains and generators are fixed, the social welfare maximization problem, which is commonly used for economic dispatch problems in power grids, reduces to a cost minimization problem with two terms in the objective function, the generator cost of generating the electricity and the energy cost of operating the trains.

With the above definitions, the overall grid-railway optimization can be posed thus:

\[
\min_{U_n \forall n \in \{1, \ldots, N\}} \sum_{n=1}^{N} \left( \sum_{d=1}^{D_n} C_n^d (P_n^d) \right) \\
+ \sum_{n=1}^{N} \frac{1}{L_n} \int_{t_n^l}^{t_n^f} P_n^l (\psi) \lambda_n (\psi) \, d\psi
\]

subject to

\[
\sum_{n=1}^{N} \left( \sum_{d=1}^{D_n} P_n^{dL} + P_n^{th} \right) + \sum_{n=1}^{N} P_n^l = 0
\]

\[
\sum_{n=1}^{N} \left( \sum_{d=1}^{D_n} P_n^{dL} + P_n^{th} \right) = 0
\]

\[
P_n^{dL} \leq P_n^L \leq \bar{P}_n^{dL}
\]

\[
m_l x_l \sin(\alpha_l(x_l)) = F_{T,l} \left( \frac{P_l}{\bar{x}_l} \right)
\]

\[
P_l (x_l) \leq P_l \leq \bar{P}_l (x_l)
\]

\[
\bar{F}_{T,l} (x_l) \leq F_{T,l} (x_l) \leq \bar{F}_{T,l} (x_l)
\]

\[
\bar{q}_l \leq \tilde{x}_l \leq \bar{q}_l
\]

\[
\bar{y}_l (x_l) \leq \tilde{x}_l \leq \bar{y}_l (x_l)
\]

\[
\tilde{t}_{l,a} (s) \leq t_{l,a} (s) \leq \bar{t}_{l,a} (s)
\]

where \(U_n\) is a decision variable set that includes \(P_n^D = \{P_n^d \forall d \in \{1, \ldots, D_n\}\}\), \(P_n^{T} = \{P_n^d (\tau_n) \forall d \in \{1, \ldots, L_n\}\}\), \(\lambda_n\) and \(\tau_n = \{\tau_n^d \forall d \in \{1, \ldots, L_n\}\}\). It is clear that the objective function in Eq. (1) is composed of the sum of the cost of the dispatchable generators (which we will denote as agents) and the energy expenses of the electric trains. These two costs are the fundamental building blocks of the two part railway dynamic market mechanism (rDMM) developed in Sections III-V. When broken down into the individual ACCs, the first term captures the railway dispatch problem with fixed loads, subject to constraints (2)-(4), which represent nodal electric power balance, nodal thermal balance and DER agent capacity limits respectively. The notation and intricacies of each constraint are fully defined in Section III. The second term corresponds to the train dispatch problem, subject to constraints (5)-(10), which represent train dynamics, electric motor power limits, traction force limits, acceleration limits, velocity limits, and schedules respectively. The notation and intricacies of each term are fully defined in Section IV.

The optimization problem as in (1)-(10) is difficult to solve, highly intractable, and poses several challenges. We note that in the decision variables in the constrained optimization problem in Equations (1)-(10) include power profiles \(P_n^D\) of the dispatchable agents, which is a standard feature in optimization problems in power systems. It should be noted that there are additional decision variables, which correspond to the power-demand profiles of trains, \(P_n^T\) and the price of electricity, \(\lambda_n\), both of which vary with the \(n\)th ACC. The final and the most important point to note is that the set of decision variables also include \(\tau_n\) which correspond to the time intervals during which the trains traverse each of the \(ACC_n\). It is clear that there is a further coupling between these additional decision variables, as the power profile \(P_n^T\) directly affects the absolute time-instances at which train \(l\) traverses all of the \(j\)th ACC, \(j \in \{n + 1, \ldots, N\}\). In addition, this coupling is nonlinear and highly complex. It is clear, however, that all these quantities, \(P_n^{dL}\), \(P_n^{th}\), \(\lambda_n\), and \(\tau_n \forall n \in \{1, \ldots, N\}\) are all decision variables that affect the overall cost and therefore have to be included simultaneously.

The second challenge introduced by the optimization problem is that its solution requires that all of the traction system agents share their private cost information \(C_n (P_n^d)\) to appropriately capture and minimize the overall system cost. This feature would likely inhibit private investment in rail-side DERs. The timescale of electric train dispatch may not individually align with that of the \(ACC_n\) dispatch. Trajectory optimization of electric railway power profiles occurs in the seconds timescale whereas energy asset dispatch is unlikely to be used at a timescale faster than 5 minutes. Most critically, the trains are entering and exiting each of the \(ACC_n\) at different times which are dependent on the pricing signal.

All of the above challenges make the problem almost intractable, since they introduce nonlinearities of various kinds, including an intricate coupling between spatial and temporal constraints. In order to make the problem more tractable, we propose a two step approach that iterates between the electric railway’s minimum cost dispatch problem with fixed train power profiles solved at each \(ACC_n\) and the train dispatch problem solved for each electric train under fixed power prices. As will become evident in Sections III - V, such a two-step approach will help decouple some of the interdependencies between various decision variables and the coupling of timescales.

III. Electric Railway Dispatch

This section describes the minimization of energy asset costs in an electric railway where the Railway Operator must guarantee that all electric and thermal constraints must be met. We will explicitly accommodate the constraints and timescales of each energy agent along the railway. Both thermal and electrical energy assets powering the electric railway are included in this optimization. Section III-A describes the different agents that operate in the system and introduces the timescales in which the agents interact. Section III-B discusses the model used for agent costs and operations within the Railway Dispatch problem. Lastly, Section III-C uses the agent costs and operations to state and solve the Railway Dispatch problem, following a similar procedure as the one developed in [15], and [16].
A. Agent Types and Timescales

Within each of the railway segments \( n \) the Railway Operator must meet the electrical and thermal demand or load during the next \( M \) future dispatch intervals which are indexed as \( K \in \{1, \ldots, M\} \). Moreover, in managing the electrical system of the railway, the Railway Operator is tasked with dispatching and compensating the electrical agents or assets within \( ACC_n \) in a way that minimizes the total cost of the operation. The electric railway dispatch problem described in this section determines optimal dispatch for each future time interval; however, the pricing and dispatch is only binding and executed for \( K = 1 \) after the Railway Dispatch problem has been solved.

We define five types of dispatchable agents within the railway power system at each node \( n \) that are classified into the following sets: heating assets (e.g. boilers), \( H_n \); electric generation assets (e.g. fuel cell, microturbines), \( E_n \); cogeneration assets (e.g. combined heat and power units), \( C_n \); storage assets (e.g. batteries), \( S_n \); low voltage side network connections (i.e. points of common coupling), \( N_n \). The set of all dispatchable agents at node \( n \) is denoted as \( A_n \triangleq H_n \cup E_n \cup C_n \cup S_n \cup N_n \).

It is assumed that within each dispatch interval \( K \) there are two faster timescales, where the first corresponds to instances \( j \in \{1, \ldots, j^{**}\} \), at each of which forecast updates of all non-dispatchable loads and generation are received. Between each \( [j, j+1) \), we introduce a faster timescale \( k \in \{1, \ldots, k^{*}\} \) where at each instance \( k \), all dispatchable agents negotiate electric and thermal generation schedules and prices (see Fig. 2). It is assumed that these timescales with \( j^{**} \) and \( k^{*} \) are such that they are nested, and that the time-intervals permit useful forecast data and sufficient negotiations, respectively.

In addition to the above agents, we also consider three types of non-dispatchable agents at each node \( n \), classified into the following sets: renewable generators, \( re_n \); electrical loads (e.g. lighting at the passenger station), \( e_n \); and thermal loads (e.g. heating of the passenger stations), \( th_n \). The set of these non-dispatchable agents at node \( n \) is denoted as \( F_n \triangleq re_n \cup e_n \cup th_n \), all of whom inject and demand electric and thermal energy from the same network as the dispatchable agents but are not dispatched by the Railway Operator. Instead, the operators in charge of each of these passive resources (i.e. the renewable asset operator for \( re_n \) and the passenger station operator for \( e_n \) and \( th_n \)) communicate the best load estimate over \( ACC_n \)’s future time intervals \( K \). This forecast update occurs every \( j \) when new updates arrive for the renewable assets, or any of the loads. The periods associated with the time scales \( k \) and \( j \) need to be such that it should accommodate new information related to the forecast and at the same time, the convergence rates of the negotiations.

Lastly, we consider electric trains \( l \in \{1, \ldots, L\} \) injecting and demanding electric power from \( ACC_n \). We denote the set of all trains at node \( n \) as \( T_n \). For the purpose of the Railway Dispatch problem, the trains are no different from the set of passive agents \( F_n \), in that the train’s electric power profile is updated for the dispatch intervals \( K \in \{1, \ldots, M\} \) at each forecast instance \( j \). However, as shown below, once we define the Train Dispatch problem faced by the train operators for each train in Section IV and the composition of the transactive controller in Section V, the train update will transform from a simple forecast update to a price-dependent minimum cost dispatch update.

B. Agent Costs and Operation

The overall goal of Railway Dispatch is to arrive at pricing information that can yield optimal setpoints for generators and utility power import. Over the time horizon of \( M \) dispatch intervals, the electric and thermal power profiles for each of the agents considered (dispatchable, non-dispatchable, and trains) is assumed to take positive (generation) and negative (load) values.

In order to capture the electric and thermal components of generation for dispatchable agents in \( A_n \) at each time period \( M \), the output of the generators is denoted by the decision variable

\[
y_i \in \mathbb{R}^M \quad \forall i \in A_n
\]

and the \( K \)’th element of \( y_i \) as \( y_{i,K} \). This decision variable maps to an electric and a thermal output as given by:

\[
g_i^e(y_i) = d_i^e y_i \quad \forall i \in A_n, \tag{12}
\]

and

\[
g_i^{th}(y_i) = d_i^{th} y_i \quad \forall i \in A_n. \tag{13}
\]

In other words, \( y_i \) is a dispatch setpoint associated with a particular electric and thermal output. For electric-only generator \( i \), the thermal conversion coefficient vector, \( d_i^{th} \in \mathbb{R}^M \), is the zero vector and the electric conversion coefficient vector, \( d_i^e \in \mathbb{R}^M \), takes positive values. We make the following assumptions regarding the constraints and costs of dispatchable agents.
Assumption 1: Electric and thermal power capacity of agent $i$ are bound by $\{P^e_{i} , P^h_{i} \}$ and $\{ P^{th}_{i} , P^{th}_{i} \}$ respectively.

Assumption 2: Capacity constraints are not binding and losses are negligible for all electrical and thermal equipment in the system other than the dispatchable agents.

Assumption 3: The cost function of each dispatchable agent $i \in A_n$ is a convex quadratic function, as is commonly derived when fitting second-order models to fuel input-power output data from generation units [21].

For a single dispatch interval $K \in \{1, \ldots , M\}$, this quadratic cost is denoted as:

$$J_i(y_i, k) = a_{i,K} + b_{i,K} y_i + \frac{1}{2} c_{i,K} y_i^2$$

and over the multi-periods as

$$J_i(y_i) = \sum_{K=1}^{M} J_i(y_i, K) \quad \forall i \in A_n.$$  

(14)

(15)

For the agents representing low voltage side network connections $N$, the cost function can be updated as a function of the equilibrium price in an external market, such as a wholesale market price. Labeling the external market price for the low voltage side network connections as $\pi_{n,j}$, the cost function parameters $a_{i,K}, b_{i,K},$ and $c_{i,K}$ in (14) are determined for $i \in N$ at each forecast instance $j$. Given that these market prices commonly represent marginal prices, the cost function parameters may be simply chosen as: $a_{i,K} = 0$, $b_{i,K} = \pi_{n,j}$, and $c_{i,K} = 0$.

Each of the passive agents in $F_n$ determine their power output for the dispatch intervals $K \in \{1, \ldots , M\}$ at each forecast instance $j$ (see Fig. 2), which are denoted by $P^e_{j,K}$ for renewable generators, $P^e_{j,K}$ for electric loads and $P^{th}_{j,K}$ for thermal loads. We drop the subscript $K$ to denote the power output vector over the dispatch intervals in $\mathbb{R}^M$ as $P^e_{j}$, $P^e_{j}$, and $P^{th}_{j}$. In order to determine the train demand, suppose that train $l$ traverses $ACC_n$ over the period $[t_{i,n-1}, t_{i,n}]$ over the dispatch interval $K$ and $P_l(t, n)$ is the corresponding forecasted demand at instance $j$, we denote this demand as $P^e_{l,j,K}$. This in turn can be summed over all $L$ trains to yield

$$\hat{P}^T_{j,K} = \sum_{l=1}^{L} P^e_{l,j,K}.$$  

(16)

For ease of exposition, we drop the subscript $K$ in (16) and simply denote the total train demand at forecast instance $j$ as $\hat{P}^T_{j}$.

C. Railway Dispatch Algorithm

With the overall costs and constraints related to all agents specified as above, we state the Railway Dispatch problem at $ACC_n$ at a fixed forecast instance $j \in \{1, \ldots , j^{**}\}$ over the dispatch intervals $K \in \{1, \ldots , M\}$ in $\mathbb{R}^M$ as:

$$\min_{y_{i,j}} \sum_{i \in A_n} J_i(y_{i,j})$$

s.t. $c_e = \hat{P}^e_j + \hat{P}^T_j + \hat{P}^{th}_j + \sum_{i \in A_n} g^e_i(y_{i,j}) = 0$  

(18)

$$c_{th} = \hat{P}^{th}_j + \sum_{i \in A_n} g^{th}_i(y_{i,j}) = 0$$

(19)

$$m^e_{i,j} = y_{i,j} - \bar{y}_{i,j} \leq 0$$

(20)

$$m^{th}_{i,j} = y_{i,j} - \bar{y}_{i,j} \leq 0.$$  

(21)

The output values for each dispatchable agent $i \in A_n$ over the dispatch interval $K \in \{1, \ldots , M\}$ at dispatch instance $j$ are denoted as $y_{i,j} \in \mathbb{R}^M$ and constitute the decision variables of the problem. These decision variables are bound at each dispatch interval $K$ by the sum of the electric loads per (18), the thermal loads per (19), the maximum capacity constraint per (20) where $\bar{y}_{i,j} = \min\{g^e_i(y_{i,j}), g^{th}_i(y_{i,j})\}$ and the minimum capacity constraint per (21) where $\hat{y}_{i,j} = \max\{P^e_{i,j}, P^{th}_{i,j}\}.$

At each forecast instance $j \in \{1, \ldots , j^{**}\}$, the power profile for each renewable generator, $P^e_{j} \in \mathbb{R}^M$, electric load, $\hat{P}^e_{j} \in \mathbb{R}^M$, thermal load, $\hat{P}^{th}_{j} \in \mathbb{R}^M$, and the total traction load, $P^T_{j} \in \mathbb{R}^M$, are updated for the dispatch interval $K \in \{1, \ldots , M\}$. The new forecasts are used to update constraints (18) and (19), with the resulting optimization problem in (17)-(21) solved again.

With each forecast update $j = j + 1$, the decision variables of the problem, denoted as $y^*_j$ optimize the cost in (17). For the dispatch intervals $K \in \{1, \ldots , M\}$ these decision variables can in turn be mapped to the electric and thermal output of the agents as $[g^e_i(y^*_i), g^{th}_i(y^*_i)] \in \mathbb{R}^M$.

The underlying optimization problem that the Railway Operator has to solve is therefore given by the solution of (17)-(21) for a given set of forecasted profiles. We propose that each $ACC_n$ solves this through an iterative negotiation process amongst the dispatchable agents within this $ACC$. This is proposed to be carried out by the faster timescales, $k \in \{1, \ldots , k^*\}$ for each $j$. That is, at each forecast instance $j$ the negotiation process starts at $k = 0$, where $P^e_{j}, P^e_{j}, \hat{P}^{th}_{j},$ and $\hat{P}^T_{j}$ are fixed, allowing the Railway Operator at $ACC_n$ to establish the electric and thermal loads for the dispatch horizon. And at $k = 0$, it is assumed that the Railway Operator for $ACC_n$ broadcasts an initial price tuple $\lambda_{n,j,k=0} = [\lambda^e_{n,j,0}, \lambda^{th}_{n,j,0}] \in \mathbb{R}^M$ consisting of electric and thermal prices.

With these initial conditions, using a Lagrangian and a gradient-based update, the decision variables $y^k_{i,j}$ and the prices $\lambda_{n,j,k} \in \mathbb{R}^M$ and $\lambda_{n,k}^{th} \in \mathbb{R}^M$ are updated at each negotiation instance $k$ as:

$$y^k_{i,j} = y^k_{i,j} - \beta y_i \left( \nabla_{y^k_{i,j}} J_i(y^k_{i,j}) + \left[ \nabla_{y^k_{i,j}} h^k \right]^T \lambda_{n,j,k}^{th} - \left[ \nabla_{y^k_{i,j}} f^k \right]^T \lambda_{n,j,k}^e \right)$$

(22)
\[
\lambda_{n,j,k+1}^e = \lambda_{n,j,k}^e + \beta_{\lambda_e}(\tilde{P}_{\tilde{r}}^e + \tilde{P}_{T}^e + \tilde{P}^e + \sum_{i \in A_n} g_i^e(y_{i,j}^e)) \\
\lambda_{n,j,k+1}^{th} = \lambda_{n,j,k}^{th} + \beta_{\lambda_{th}}(\tilde{P}_{\tilde{r}}^{th} + \sum_{i \in A_n} g_i^{th}(y_{i,j}^{th}))
\]

(23)

(24)

where \( \beta_{\lambda_e}, \beta_{\lambda_{th}} \in \mathbb{R} \) and \( i \in A_n \) are positive step-size parameters and \( \lambda_{n,j,k} \in \mathbb{R} \) are the penalty for violating capacity constraints (20) and (21). The penalty function updates are given by:

\[
\mu_{i,j}^{k+1} = \max\{0, \mu_{i,j}^{k} + \beta_{\mu} \lambda_{i,j}^{k+1}\}
\]

(25)

where \( \beta_{\mu} \) is a positive step-size parameter.

It is assumed that these iterations occur at each \( k \) and converge as \( k \to k^* \) for some \( k^* \). As outlined in [14], under suitable convexity conditions, it can be shown that convergence to unique optimal values takes place.

Defining \( \tilde{P}_r^e = [\tilde{P}_{\tilde{r}}^e, \tilde{P}_r^e, \tilde{P}_{\tilde{r}}] \) as the estimate of the fixed assets at \( j \), these estimates can be updated with \( j \) as the new forecasts arrive. The forecast update enables an improved dispatch of the agents across the dispatch intervals \( K \in \{1, \ldots, M\} \).

Note that in practice, exit conditions based on the agent output and price profile updates within the negotiation process can be established such that the forecast update is promptly initiated. These exist conditions follow the form: \( |y_{i,j}^{k+1} - y_{i,j}^{k}| \leq \gamma_{k} | \lambda_{n,k+1,j} - \lambda_{n,k,j} | \leq \gamma_{k}^2 \). If these conditions are met, the equilibrium agent output and price profiles can be set as \( y_{i,j}^* = y_{i,j}^{k+1} \in A_n \) and \( \lambda_{n,j}^* = \lambda_{n,k,j} \), respectively.

Similarly, exit conditions can be established for the forecast update process, such that the agent output and price profile updates can be communicated for dispatch and settlement if so desired. These exist conditions follow the form: \( |y_{i,j}^{k+1} - y_{i,j}^{k}| \leq \gamma_{k} | \lambda_{n,j+1,i} - \lambda_{n,j,i} | \leq \gamma_{k}^2 \). If these conditions are met, dispatch for the first interval \( K = 1 \) can be established using agent output profiles \( y_{i,j}^* = y_{i,j+1}^{k+1} \forall i \in A_n \) and price profiles \( \lambda_{n,i}^* = \lambda_{n,j+1,i}^* \).

In summary, the Railway Dispatch algorithm for \( K = 1 \) starts at \( k = 1, j = 1 \) with a forecast of the power profile for each renewable generator, \( \tilde{P}_r^e \in \mathbb{R}^M \), electric load, \( \tilde{P}_r \in \mathbb{R}^M \), thermal load, \( \tilde{P}_{\tilde{r}}^{th} \in \mathbb{R}^M \), and returns the optimal agent output profiles \( y_{i,j}^* \), the total traction demand profile \( P_T^* \) and the price profiles \( \lambda^* \) that can be used for dispatch and settlement of dispatch interval \( K = 1 \). The dispatch interval window then shifts over, with \( K = 2 \) corresponding to the active dispatch interval and the process repeats.

**IV. TRAIN DISPATCH**

In the previous section, we assumed that at each \( ACC_n \), the train loads \( \mathcal{T}_n \) were fixed, and the dispatch of the active agents \( A_n \) was optimized. In this section, we address the fact that these train loads are indeed flexible, and pose a constrained optimization problem for determining the optimal profiles of power generation and consumption. In Section IV-A, we describe the physical model of the train. Next, we define the cost minimization problem for a track with multiple pricing regions, managed by \( ACC_n \) in Section IV-B. With this Train Dispatch accomplished, in Section V, we describe how the Railway Dispatch optimization described in Section III can be stitched together with the Train Dispatch optimization to result in an overall transactive control framework for the combined grid-railway infrastructure. This overall framework is validated using numerical simulation of the Amtrak NEC in Section VI.

**A. Dynamic Model of Electric Trains**

The power consumption of the electric train depends on the traction force, which in turn depends on the overall train dynamics. In this section, we derive the underlying dynamic model of the train and the corresponding power consumption profile. The position, velocity and acceleration of train \( i \) in the direction of motion \( i \) are denoted by \( x_i, \dot{x}_i, \ddot{x}_i \) respectively. We proceed by defining the three forces with a component in the direction of motion of the train \( i \) as it travels at an angle \( \alpha_i(x_i) \) from the horizon. The gravitational force on the train can be decomposed into the direction of motion \( i \) as \(- m_i g \sin(\alpha_i(x_i)) \) and the direction normal to the ground \( j \) as \(- m_i g \cos(\alpha_i(x_i)) \), where \( m_i \) is the total mass of the train. The electric motors converting electrical into mechanical power are assumed to result in the traction force \( F_{T,i} \) in the \( i \) direction. An opposing force \( F_{DF,i} \) is also included in the model, which includes a drag force and a friction force [22].

With these definitions, we can derive the equation of motion in the direction of motion \( i \) as

\[
m_i \ddot{x}_i = F_{T,i} - F_{DF,i} - m_i g \sin(\alpha_i(x_i)).
\]

(26)

The traction force \( F_{T,i} \) is a function of both the electric power \( P_i \) and \( \dot{x}_i \), and in general a mapping of the ratio \( P_i/\dot{x}_i \). The drag-friction force can be represented as

\[
\sum F_{DF,i} = A_i + B_i \dot{x}_i + C_i \dot{x}_i^2.
\]

(27)

known as Davis Equation [22]. This industry standard approximation captures the rolling resistance effect at low speed through the linear term and the drag force through the quadratic term. More detailed drag-friction models for train cars can be found in [23].
B. The Constrained Optimization Problem

Before stating the optimization problem, a few notations and details related to the train trajectories are addressed. Let us assume that train $l$ departs location $x_0$ at time $t_0$ and arrives at a final destination $x_f$ at time $t_f$. Let us assume that the train stops at passenger stations denoted by $s \in \{0, \ldots, D\}$ between $t \in [t_{l,a}(s), t_{l,a}(s)]$, where $t_{l,a}(s)$ and $t_{l,a}(s)$ are the arrival and departure times, respectively, from station $s$ and $x_{l,s}$ denotes the position of station $s$. Over the period $[t_0, t_f]$, the train is assumed to traverse $n$ subsections, each dispatched by ACC$_n$, with the travel during the time-interval $[t_{l,n-1}, t_{l,n}]$ which corresponds to a position interval $[x_{n-1}, x_n]$, and not necessarily coincident with the station locations. Let us denote the corresponding power-demand profile for this train as

$$P_l(t) = \begin{cases} P_l(t, 1) & t \in [t_{l,0}, t_{l,1}] \\ \vdots \\ P_l(t, N) & t \in [t_{l,N-1}, t_{l,N}]. \end{cases} \quad (28)$$

We assume that the railway is level at the passenger stations (i.e. $\alpha(x_{l,s}) = 0$, $\forall s \in \{0, \ldots, D\}$) implying that the power demand of the train during the stop is equal to zero, $P_l(t) = 0 \in [t_{l,a}(s), t_{l,a}(s)]$, $\forall s \in \{0, \ldots, D\}$.

During the same time-intervals, the train operator $l$ faces prices $\lambda_l(t, n) \forall t \in [t_{l,n-1}, t_{l,n}]$, $\forall n \in \{1, \ldots, N\}$, summarized as

$$\lambda_l(t) = \begin{cases} \lambda_l(t, 1) & t \in [t_{l,0}, t_{l,1}] \\ \vdots \\ \lambda_l(t, N) & t \in [t_{l,N-1}, t_{l,N}]. \end{cases} \quad (29)$$

We determine the prices $\lambda_l(t)$ from the price profiles $\lambda^*_n$ determined in Section III as follows. Suppose that the time interval $[t_{l,n-1}, t_{l,n}]$ is specified, which corresponds to the time-interval over which train $l$ traverses ACC$_n$. We choose an arbitrary time $t_K \in [t_{l,n}, t_{l,n+1}]$, and suppose it corresponds to the $K$'th interval (for example, the time horizon corresponds to $K = 1$ in the example shown in Fig. 2). Then $\lambda_l(t, n)$ corresponds to the $K$'th element of $\lambda^*_n \in \mathbb{R}^M$. The same process is repeated for all $n = 0, \ldots, N$.

With the above definitions in place, the train dispatch problem, which corresponds to the optimal profiles of generation and consumption of power corresponding to each train is now posed: Assuming that $\lambda_l(t)$, defined in (29), is determined using the Railway Dispatch algorithm in Section III-C, the train power profiles are determined through the minimization of the energy cost of train $l$ as

$$\min_{x_1, x_f} \int_{t_0}^{t_f} P_l(\psi) \lambda_l(\psi) d\psi \quad (30)$$

subject to

$$m_l \dot{x}_l + F_{DF,l}(\dot{x}_l) + m_l g \sin(\alpha_l(x_l)) = F_{T,l} \left( \frac{P_l}{\dot{x}_l} \right) \quad (31)$$

$$P_l(x_l) \leq P_l \leq \bar{P}_l(x_l) \quad (32)$$

$$F_{T,l}(\dot{x}_l) \leq F_{T,l} \leq \bar{F}_{T,l}(\dot{x}_l) \quad (33)$$

$$a_t \leq \dot{x}_l \leq \bar{a}_l \quad (34)$$

$$\underline{\omega}(x_l) \leq \dot{x}_l \leq \bar{\omega}(x_l) \quad (35)$$

$$t_{l,a}(s) \geq t_l(s), \quad s \in \{0, \ldots, D\} \quad (36)$$

$$t_{l,d}(s) \leq \bar{t}_l(s), \quad s \in \{0, \ldots, D\}. \quad (37)$$

In the above, (30) represents the total energy cost incurred by train $l$ over $[t_0, t_f]$. Eq. (31) corresponds to the train kinematics, which includes the dependence of the traction force on the power-profile as well as the train velocity. Inequalities (32)-(37) denote various constraints on power, force, velocity, acceleration, and times that need to be accommodated. Limits in (33) are determined by the traction force curve of the train manufacturer, limits in (34) and (35) correspond to safety considerations and civil speed limit restrictions in densely populated areas. Limits in (36) and (37) correspond to schedule constraints.

The optimization problem in (30)-(37) continues being nonlinear and nonconvex due to the fact that $P_l(t)$ in (30) is a highly nonlinear function of the decision variables $(x_l, \dot{x}_l)$, as it depends on the traction force and therefore $\dot{x}_l$. It should be noted that all of the time-intervals $[t_{l,i}, t_{l,i+1}]$ also depend on the decision variables, which adds to the nonlinearity as well. However, the crucial difference between the solution of this problem and the overall problem stated in Section II is the decoupling of the prices from the decision variables $(x_l, \dot{x}_l)$ and therefore the time-instants $t_{l,i}$. As the prices in turn are determined from both a grid-wise and a railway-wise perspective as outlined in Section III, decoupling them from this optimization problem as outlined above simplifies the problem significantly. In particular, as outlined above, the price profiles $\lambda_l(t)$ can be determined using the Railway Dispatch algorithm for a given time-interval, and the time-interval in turn is determined using the optimization procedure of the Train Dispatch outlined here in this section in (30)-(37). It is this decoupling into these two separate Dispatch problems that represents the main contribution of this paper.

The solution of the Train Dispatch problem in (30)-(37) can be determined using any one of several commercial software packages such as Matlab’s fmincon [24]. Numerical solvers need to be employed to transform the continuous variables (including the decision variables) to discrete-time variables of length $(t_f - t_0) \delta\psi$, where $\delta\psi$ is the time step.
V. OVERALL TRANSACTIVE CONTROL ARCHITECTURE - RAILWAY DYNAMIC MARKET MECHANISM (rDMM)

In Section II, we posed a combined optimization problem of Railway Dispatch and Train Dispatch in the form of (1)-(10). In order to make the problem more tractable, we divided it into two steps, which were addressed in Sections III and IV. In Section III, we presented a constrained optimization problem that solves for the optimal dispatch of energy assets, where loads including those of trains as well as renewable generation were assumed to be fixed and the electric and thermal schedules and prices for the generators along the track are determined as $[g^f_i(y^*_l), g^{th}_i(y^*_l)] \in \mathbb{R}^M$ and $\lambda^*_n = [\lambda^c_n, \lambda^{th}_n] \in \mathbb{R}^M$ respectively for dispatch intervals $K \in \{1, \ldots, M\}$ respectively in (17)-(21). This formulation captures the agent operational cost in the first term of (1) and ensures that the agent constraints (2)-(4) are met. In Section IV, we focused on the optimization of the train power consumption profiles themselves. In particular, we showed how each train operator can solve a constrained optimization problem in (30)-(37) to minimize energy costs given the track prices and required schedule. Minimizing the sum of trains’ energy costs is equivalent to the second term of (1), subject to the constraints (5)-(10) for each train.

In this section, we summarize how the results of Sections III and IV can be interleaved to solve the combined optimization problem posed initially in Section II. This is accomplished by adjusting the power demand of the trains as a function of the prices from the Railway Dispatch problem, as a form of automated Demand Response at fast time-scale, also referred to as transaction control [25], [26]. This transaction control-based $ACC_n$ price provides an incentive for the electric trains to modify their dispatch by iteratively solving the cost minimization problem (30)-(37) to minimize energy costs given the track prices and required schedule. In what follows, we describe the overall transactive control architecture (see Fig. 4 for an overall schematic and Algorithm 1 for details).

The top block in Fig. 4 denotes the Agent Dispatch, and corresponds to the following functionality: at each forecast instance $j \in \{1, \ldots, j^*\}$, the Agent Dispatch block takes as inputs the traction power demand $P^T_j$, passive agent output curves $\hat{P}^T_j$ and the energy prices from the low voltage side network connections $\pi^N_{n,j}$, and returns the price profiles $\lambda^*_{n,j}$ and the Railway Dispatch profiles $y^*_l$. As mentioned in Section III, the energy prices from the low voltage side network connections are used to determine the cost function of the dispatch problem (14). Next, these cost curves are used to update the objective function (17) and the traction and passive agent output curves are used to update constraints (18)-(19). The iterative dynamics in (17)-(21) are solved using (22)-(25) for negotiation instances $k \in \{1, \ldots, k^*\}$, stopping when the negotiation exit conditions defined earlier in Section III are met at each $ACC_n$.

The block on the right in Fig. 4 corresponds to the functionality of the Train Operator, who receives the price of energy profile for each train $l \in \{1, \ldots, L\}$ in (29) using the electricity portion of the price profiles from the Agent Dispatch block, by determining the corresponding dispatch interval $K$ and the corresponding element of the price vector $\lambda^*_n$. These are then used in the Train Dispatch block (bottom) to determine the next dispatch profile forecast for each train $P^*_l$.

The bottom block in Fig. 4 denotes the Train Dispatch and uses the prices $\lambda_i(t)$ as in (29) to determine the cost-minimizing train dispatch for every train. This is accomplished using the optimization procedure discussed in Section IV: the energy profile from the Agent Dispatch block are used to update the objective function of the Train Dispatch problem in (30). Once this update is complete, a new power demand profile of the train $P^*_l$ is determined by solving (30)-(37).

The block on the left in Fig. 4 represents the Railway Operator, who collects the new power demand profiles for each of the trains $l \in \{1, \ldots, L\}$ after each forecast instance $j$ and assembles the total traction power demand for each $ACC_n$ in (16). This new profile is used in the negotiations (22)-(25), in addition to new forecasts that may become available from other passive agents $\hat{P}^T_j = [\hat{P}^T_{j1} + \hat{P}^T_{j2}, \hat{P}^T_{j3}]$ at each $j$ and energy prices from the low voltage side network connections $\pi^N_{n,j}$ used to determine the cost function parameters $\alpha_{i,K}, \beta_{i,K}$, and $c_{i,K}$ in (14) for $i \in N$.

The cycling between the Agent Dispatch and Train Dispatch blocks repeats for forecast instances $j \in \{1, \ldots, j^*\}$. If the resulting price $\lambda^*_{n,j}$ and Railway Dispatch $y^*_l$ profiles meet the forecast exit conditions for the network $(y^*_l - y^*_{l+1}) \geq \gamma^y_j$ and $|\lambda^*_{n,j+1} - \lambda^*_{n,j}| \geq \gamma^\lambda_j$, then the algorithm stops, dispatching the agents at the last negotiation equilibrium $y^*_l$ and are compensated based on the price profiles $\lambda^*_n$ from the top block. Similarly, trains are dispatched based on the last forecast update $P^*_l$ from the bottom block and train operators are required to pay the last negotiation equilibrium price $\lambda^*_n$. As mentioned before, the overall iteration is ensured to stop by using suitable exit conditions. A settlement procedure may be designed to collect...
the payments of the agents at a slower frequency than the convergence of the \( rDMM \) (i.e., monthly payments). Once dispatch takes place for \( K = 1 \) the dispatch horizon shifts by one interval, and the procedure set forth with \( j = 1 \) starts again.

**Algorithm 1** Railway Dynamic Market Mechanism \( rDMM \)

| Train Inputs: \( m_l, F_{DF,l}, \alpha_l, F_{T,l}, P_l, T_l, \hat{P}_l, \hat{F}_{DF,l}, \hat{m}_l, v_l, \hat{v}_l, (s), \hat{T}_l(s) \forall l \in \{1, \ldots, L\}, s \in \{0, \ldots, D\} \) |
| Agent Inputs: \( P_{l}^{e}, P_{l}^{th}, P_{l}^{th}, J_{l}(y_{l}), \pi_{n,j}^{N}, \pi_{n,j}^{T} \forall l \in \mathcal{A}_n \) |

\( j = 1; \lambda_{n,0} = 0 \)  
{Dispatch Interval \( K = 1 \)}  
\( \text{while } |y_{n,j}^* - y_{n,j-1}^*| > \gamma_{j}^l \text{ OR } |\lambda_{n,j}^* - \lambda_{n,j-1}^*| > \gamma_{j}^\lambda \text{ do} \)  
for \( l \in \{1, \ldots, L\} \) do  
  Update (29) using \( \lambda_{n,j}^l \)  
  Solve (30)-(37) for \( \hat{P}_l^e \)  
end for  
{Forecast instance update across each \( ACC_{n} \)}  
for \( n \in \{1, \ldots, N\} \) do  
  Update (16) using \( P_{l}^{e} \forall l \in \{1, \ldots, L\} \)  
  Update (17)-(19) using \( P_{j}^{th}, \hat{P}_j^{th}, \pi_{n,j}^{N} \)  
  \( k = 0 \)  
  {Agent negotiation}  
  \( \text{while } |\lambda_{n,k+1} - \lambda_{n,k}^*| > \gamma_{h}^k \text{ do} \)  
  Solve for \( y_{l,k+1}^* \) in (22)  
  Solve for \( \lambda_{n,k+1}^l \) in (23) and \( \lambda_{n,k+1}^{th} \) in (24)  
  \( k++ \)  
end while  
  Update \( \lambda_{n,j} = \lambda_{n,j+1}^* \) and \( \lambda_{n,j+1} = \lambda_{n,k}^* \)  
end for  
\( j++ \)  
end while  
Dispatch \( \lambda_{n,j}^{e}, y_{l,k}^{e}, P_{l}^{T} \) for \( K = 1 \).

### VI. SIMULATIONS

#### A. Methods and Parameters

The northern Amtrak NEC between Boston, MA and New Haven, CT (within the ISO-NE power system) emerges from a review of the electric railway systems in the United States as a prime case study for our analysis, due to its four segmented rail power zones that result in the pricing regions identified in Fig. 5. The four area control centers, \( ACC_{n} \) identified with \( n \in \{1, 2, 3, 4\} \) are powered by the substations at Sharon, MA; New Warwick, RI; London, CT; and Branford, CT respectively and are considered separate pricing regions, each with price \( \pi_{n} \forall n \in \{1, 2, 3, 4\} \).

Using the location of these substations, publicly available electric utility tariff information [29]–[31] and real-time energy market data from ISO-NE [32], we estimate the total energy cost to the operator (composed of supply and delivery charges) at each one of the four \( ACC_{n} \) which are in turn used as the low-voltage side network connection costs \( \pi_{n,j}^{N} \) of the four network connection agents \( \mathcal{N}_{n} \) that are used to update the quadratic cost curve of these agents in (15). Based on the characteristics of the Route 128/University Park (\( ACC_{1} \)) and Providence (\( ACC_{2} \)) passenger stations and their surrounding commercial spaces we also add cogeneration assets \( C_{n} \) and boilers \( \mathcal{H}_{n} \) using NREL’s System Advisory Model (SAM) [33] for thermal \( P_{l}^{th}, \hat{P}_l^{th} \) and electric \( P_{j}^{th}, \hat{P}_j^{th} \) sizing, and estimation of the quadratic cost function coefficients in (15) as summarized in Table 1.

Due to the large roofs and parking lots near the University Park, Providence and New Haven passenger stations, we also assumed that three PV solar arrays \( r_{c_{n}} \) can be added at \( ACC_{1}, ACC_{2} \) and \( ACC_{4}, \) and used SAM alongside satellite imagery to size the arrays and estimate their production, \( P_{l}^{c_{n}} \). We also used the satellite imagery to measure the footprint of the passenger stations and estimate the electric \( P_{j}^{c_{n}} \) and thermal \( (P_{l}^{th}) \) load using the EPA commercial building templates that can be accessed through SAM.

Amtrak’s high-speed Acela Express service along the NEC utilizes high-speed locomotives developed by Bombardier in the late 1990s based on the French TGV [34]. Acela Express trains have a total empty weight of 531.2\( MT \) and a full capacity weight of 556.7\( MT \). In simulation we assume a partially occupied weight of \( m_l = 545MT \). Although the

![Fig. 5](image)

**Fig. 5**: Map of the four pricing regions identified along Amtrak’s NEC between University Park Station in Massachusetts and New Haven Station in Connecticut. This graphic and the road slope profile, \( \alpha_l(x_l) \), used in simulation were developed using Google Earth Pro [27] and the data collected using the GPS of a mobile phone and the MyTracks iOS application [28].

![Fig. 6](image)

**Fig. 6**: Schematic of the Area Control Centers proposed for Amtrak’s northend NEC used in simulation. The four \( ACC_{s} \) have varying levels of load, renewable deployment, dispatchable agents and network energy pricing.
Acela trains are designed to achieve a 264 km/h top speed, they are limited in operation to 240 km/h which is equivalent to $66.7 m/s$.

The maximum train traction power $P_T$ is 9.2 MW, while regenerative braking is limited in operation to $P_B = -6.0 MW$ [1]. In the absence of public data on the acceleration rates of high-speed trains like the Acela, the estimates used in our simulation ($a_1 = -0.5 m/s^2$ and $a_2 = 0.5 ms^{-2}$) were adopted from models of electric train systems used by EPRail. Fitting the Davis equation (27) to the Acela Express drag and rail friction curve, $F_{DF, i,j}$, we have that $A_1 = 10,195.16$, $B_1 = 65.81$, and $C_1 = 25.02$.

Using the pricing information of the nodes along the track and the Amtrak Acela Express train timetable [35] as departure $(t_d(s))$ and arrival $(t_a(s))$ times for each station, we simulate a train following a power profile that minimizes total work as a baseline and a train dispatched by the rDMM methodology summarized in Algorithm 1.

### B. Results

We now report the results obtained using the rDMM outlined in Section V, which we will denote as the transactive controller. In particular Algorithm 1 was run with all numerical parameters as in Table I. The results are shown in Fig. 7 for a single train’s travel profile corresponding to for the 6:21AM University Park departure of Acela 2155 on January 18, 2018, a day that exhibited large network pricing differentials. These results include the energy prices $\lambda_i$ as the train traverses the four ACCs, and the position and velocity profiles of the train. It can be seen that the train schedules are met, and the network pricing varies significantly from a day that exhibited large network pricing differentials. These results also show that the train satisfies the maximum speed limit restrictions along the route, set by the low voltage side network connection cost, and the velocity limits are accommodated. The most interesting result corresponds to the energy price shown in the top plot in Fig. 7, and corresponds to the minimization of the cost function in Eq. (17). This price profile in turn leads to an optimized cost in Eq. (30) of $200.62 for this single train travel.

\[
\begin{align*}
ACC_{10} & \quad Agent & \quad P^{e,th}_i & \quad d^e_i & \quad d^{th}_i & \quad b_{i,K} \\
1 & \quad \mathcal{H}_1 & \quad 10,432 & \quad 0 & \quad 1 & \quad 0.0303 \\
1 & \quad C_1 & \quad 1,550 & \quad 1 & \quad 1.02 & \quad 0.0629 \\
1 & \quad N_1 & \quad 10,000 & \quad 1 & \quad 0 & \quad \pi_{1,j} \\
2 & \quad \mathcal{H}_2 & \quad 20,864 & \quad 0 & \quad 1 & \quad 0.0303 \\
2 & \quad C_2 & \quad 4,560 & \quad 1 & \quad 2 & \quad 0.0818 \\
2 & \quad N_2 & \quad 10,000 & \quad 1 & \quad 0 & \quad \pi_{2,j} \\
3 & \quad \mathcal{H}_3 & \quad 10,000 & \quad 1 & \quad 0 & \quad \pi_{3,j} \\
4 & \quad N_4 & \quad 10,000 & \quad 1 & \quad 0 & \quad \pi_{4,j}
\end{align*}
\]

TABLE I: Agent numerical parameters used in simulation. Maximum capacity is expressed for the binding thermal or electric characteristic and can be identified by the mapping coefficient $d^e_i$ or $d^{th}_i$ that is equal to 1. The other cost function parameters in (14) were used at a minimum in simulation, setting $c_i$ to a small positive quantity.

### C. Discussions

We now make some observations based on the results obtained above. We evaluate the optimality of the proposed transactive architecture, through a comparison with two other train profiles which are shown in Fig. 7. The first one corresponds to a field dataset that was collected using the GPS of a mobile phone and the MyTracks iOS application [28] on the Acela train. It can be seen that the maximum speed of $66.7 m/s$ as well as the position datasets corresponding to the transactive controller are consistent with our field dataset. We computed the corresponding train cost for this field dataset assuming the same dynamical model for the Acela train employed in simulation, summarized in Equation (31), which allows us to solve for the traction force $F_{T, i,j}$ as a function of the position, velocity and acceleration datasets collected with the phone GPS and accelerometers. This force dataset can in turn be used in conjunction with the volatility dataset to arrive at the transactive power profile for the route. Finally, the marginal cost along the route, set by the low voltage side network connection cost, can be applied to the profile to determine the total cost. It was observed that this cost was $865.01, which shows that our rDMM results in a 75% reduction. It is possible that the actual reduction from the rDMM may be somewhat smaller, as we have not incorporated other speed limit restrictions along the track such as rail crossings and densely populated areas in the simulation of the rDMM.

The second profile shown in Fig. 7, denoted as minimum work, was obtained by solving the Train Dispatch problem in (30)-(37) with a uniform price profile $\lambda_i(\tau) = 1$. This yielded a position and velocity profile as shown in Fig. 7. The corresponding price profile is shown in Fig. 7 as well, which led to a total trip cost $273.69. Note that the train dispatched by the rDMM methodology can achieve a 25% cost reduction when compared to the train dispatched under the standard minimum work (from $273.69 to $205.62), providing an initial estimate of the value of incorporating dynamic, price-responsive train dispatch in the electric railway operation.

In summary, the numerical simulation of the proposed two-step optimization mechanism applied to a segment of the Southbound Amtrak service along the NEC resulted in a 25% reduction in energy costs when compared to a standard trip optimization based on minimum work, and a 75% reduction in energy costs when compared to the train cost calculated using a field dataset. To our knowledge, the current procedure adopted by Amtrak for the train profile does not have such an optimization approach, but rather allows train operators to accelerate and decelerate the train at their discretion with the supervision and intervention of the Positive Train Control system. The approach in [8], implemented in freight locomotives, employ algorithms similar to the Minimum Work method reported above.

### VII. Conclusions and Future Works

Electric trains are a major untapped source of demand-side flexibility in electricity networks. Our findings contribute to the evolution of transportation control systems devoted to work minimization toward higher-level objectives such as the social welfare maximization of joint transportation-electric infrastructures. In particular, our proposed 2-step optimization of railway dispatch of all DER agents along the track followed by train dispatch, facilitated by coordinated operations
also incur demand or capacity charges related to costs ($\text{/MW}h$). In reality the Railway Operator will also incur demand or capacity charges ($\text{$/MW}$) from the utility or ISO. These charges can also be reduced, in principle, using a transactive control methodology where the incentive signal shifts and smooths the power profile of the individual trains such that a reduction in demand at the main interconnection (traction substation) is met. We considered including the demand charge component within the cost function of low voltage side network connections $N_n$ as a second order term, but this methodology did not appropriately capture the time-scale (typically months) at which demand charges are evaluated. A means of achieving demand charge management is to update constraints within the optimization problem, modifying the minimum and maximum power limits $P_l(x_1)$ and $P_u(x_1)$ in (32).

2) **Regulation and Reserve Market Participation:** Similar to the research direction regarding demand charges, we would like to extend the services provided by the transactive control system to the electrical network beyond energy and onto products for frequency regulation and operational reserves. Practical limitations of providing these services as well as the incentive and compensation mechanisms remain to be explored.

3) **Mass Transit Systems:** Our analysis focused on the practical intricacies of high-speed rail systems. Although our simulations were based on the high-speed rail example in the United States, the Amtrak Acela service, we are aware that other systems, such as the MBTA mass transit T service also evidences discontinuities in energy price along their track and have strategic plans to add rail-side generation [36]. Extending our simulation work to mass transit systems would widen the applicability of our proposed transactive control architecture.

4) **Stochastic Modeling:** In this paper, we have employed a deterministic approach, and have focused on the complex, nonlinear relationship between key decision variables that change with space and time. An extension to consider a stochastic counterpart of the approach proposed, building from the agent dispatch component, which requires energy market price, renewable generation and load forecast inputs at each forecast instance $j = \{1, \ldots, j\}$, as well as the railway dispatch component outputs (see Fig. 4).

**REFERENCES**


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